

CURSO AVANZADO DE GEOMETRÍA

BENJAMIN BODE

In this course the students will learn the definitions and most common techniques of symplectic geometry, preparing them for a level of independent study and research in geometry. Since its beginnings as the Hamiltonian formulation of classical mechanics, symplectic geometry has developed to an essential part of modern geometry with connections to many areas of geometry, topology and mathematical physics.

Starting from the fundamental definitions and theorems by Darboux and Moser, the course will cover Hamiltonian dynamics, providing the students with the historically most important application of symplectic geometry. In the later part of the course, we will also discuss the relations to contact geometry and complex geometry via Kähler manifolds. Towards the end of the course, we give the basic idea behind the proof of the weak version of Arnold's conjecture using Floer homology.

Throughout the course there will be assignments and examples, providing concrete cases of the general theory.

Below is an outline of the concepts and theorems covered in this course.

1. Symplectic vector spaces, cotangent bundles, symplectic forms, symplectic manifolds and submanifolds (~ 2 weeks)
2. Symplectomorphisms, Darboux theorem, Moser's trick, (~ 3 weeks)
3. Hamiltonian dynamics, momentum maps, Poisson brackets, symplectic reduction (~ 3 weeks)
4. Contact manifolds, Legendrian submanifolds, symplectization (~ 2 weeks)
5. Compatible complex and almost-complex structures, Kähler manifolds (~ 3 weeks)
6. Arnold conjecture about fixed points of Hamiltonian diffeomorphisms, Floer homology (~ 2 weeks)

The course does not follow one particular textbook. The list below (in alphabetical order) is a collection of useful references.

REFERENCES

- [1] M. Audin, M. Damian: *Morse theory and Floer homology*. Springer Verlag (2014).
- [2] A. Cannas da Silva: *Lectures on symplectic geometry*. Springer Verlag (2008) (available at <https://people.math.ethz.ch/~acannas/Papers/lsg.pdf>)
- [3] J. Etnyre: *Introductory lectures on contact topology*. Proceedings of Symposia in Pure Mathematics. **71** (2003): 81–107. (available at arXiv:math/0111118)
- [4] H. Geiges: *An introduction to contact topology*. Cambridge University Press (2008).
- [5] D. McDuff, D. Salomon: *Introduction to symplectic geometry*. 3rd edition, Oxford University Press (2017).

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