

## PROGRAMO CURSO AVANZADO DE ÁLGEBRA

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### REPRESENTATION THEORY OF FINITE GROUPS

The goal of the course is an introduction to the representation theory of finite groups which aims to understand the structure of a group through its actions on linear structures, in particular vector spaces. Representation theory has contributed to the understanding of the structure of groups right from its beginning, but has since then also developed into an elegant theory of its own which still offers many puzzling questions. In this course we will start by the basic facts on ordinary representation theory, i.e. the action on vector spaces over fields of characteristic 0, and later develop the theory also in the case of positive characteristic. Finally, at the end of the course we will connect these two approaches through evaluation rings.

The topics in this course will include:

- (1) Semisimple algebras, simple and indecomposable modules, Theorem of Jordan-Hölder, Theorem of Wedderburn-Artin
- (2) Representations of groups, group rings, Maschke's Theorem, characters and character tables, Burnside's  $p^a q^b$ -Theorem
- (3) Radicals and socles, the number of simple  $KG$ -modules, the Krull-Schmidt Theorem
- (4) Tensor products, induced modules, projective modules, relatively free and projective modules, Frobenius reciprocity, Mackey decomposition
- (5) Vertices and sources, Green correspondence
- (6) Representation type, projective hulls, basic Clifford Theory, group algebras with finite representation type
- (7) Defect groups and their basic properties
- (8) Evaluation rings,  $p$ -modular systems, orders and lattices, the Cartan-Brauer triangle, decomposition and Cartan matrix
- (9) Some open problems on representations of groups

### LITERATURE

There are many books on the topic. Some of those which can be useful to this course are:

For ordinary representation theory:

- Gordon James and Martin Liebeck, *Representations and characters of groups*, Cambridge University Press 1993, 2001
- I. Martin Isaacs, *Character Theory of Finite Groups*, Academic Press, 1976

For modular representation theory:

- J.L. Alperin, *Local Representation theory*, Cambridge University Press, 1986, 1993

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- Gabriel Navarro, *Characters and blocks of finite groups*, Cambridge University Press, 1997

An extensive book:

- Charles W. Curtis and Irving Reiner, *Methods of Representation Theory. With applications to finite groups and orders*, John Wiley and Sons, 1981