MAXIMUM PRINCIPLES AND GEOMETRIC APPLICATIONS

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Abstract

The *Omori-Yau maximum principle* is said to hold on an n-dimensional Riemannian manifold M if, for any smooth function $u \in \mathcal{C}^2(M)$ with $u^* = \sup_M u < +\infty$ there exists a sequence of points $\{p_k\}_{k\in\mathbb{N}}$ in M with the properties:

$$\text{(i)} \quad u(p_k) > u^* - \frac{1}{k}, \quad \text{(ii)} \quad \|\nabla u(p_k)\| < \frac{1}{k}, \quad \text{and (iii)} \quad \Delta u(p_k) < \frac{1}{k}.$$

In this sense, the classical result given by Omori (1967) and Yau (1975) states that the Omori-Yau maximum principle holds on every complete Riemannian manifold with Ricci curvature bounded from below. A *weaker* form of the maximum principle is obtained by dropping condition (ii) above from the requirements on the sequence $\{p_k\}_{k\in\mathbb{N}}$. That is, the *weak maximum principle* is said to hold on M if, for any smooth function $u \in \mathcal{C}^2(M)$ with $u^* = \sup_M u < +\infty$ there exists a sequence $\{p_k\}_{k\in\mathbb{N}}$ in M satisfying (i) and (iii) above. It is also known that the fact that the weak maximum principle holds on M is equivalent to the *stochastic completeness* of the manifold. In particular, the weak maximum principle holds on every parabolic Riemannian manifold. The aim of this lecture is to give an introduction to the Omori-Yau maximum principle, starting from its classical formulation up to the most recent generalizations, and to introduce some of its applications to differential geometry.

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