

Kurusch Ebrahimi-Fard: Multiple Zeta Values and Hopf algebras

Since Euler's famous work, classical zeta values play an important role in number theory. More recently, iterated sums of similar type, so-called multiple zeta values (MZVs) and their generalizations, moved into the limelight. They have been studied intensively leading to connections with arithmetic and algebraic geometry, algebra, combinatorics and computer sciences. Interestingly, these numbers play a crucial role as coefficients in expansions of Feynman amplitudes arising in perturbative quantum field theory.

MZVs can be represented in terms of Chen's iterated integrals of rather simple rational functions. Many algebraic relations among MZVs, called double shuffle relations, arise from the interplay between the sum and the integral representations. The resulting complex patterns have a combinatorial flavor, and can be encoded in terms of shuffle-like algebras, Lie algebras, and Hopf algebras. Conjecturally, all algebraic relations among MZVs can be obtained this way. Therefore, one of the main activities in the field is to understand the structure of the algebra of MZVs, and their generalizations.

In the light of recent intensive research on (pre-)Lie and Hopf algebras in combinatorics, a natural question would be to explore the ideas outlined in Hoffman's Oberwolfach Reports [3]. Good introductory references on the theory of MZVs and related aspects are available, see e.g. [1,2]. More advanced references are [4,5,6].

References:

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- [5] T. Terasoma, "Geometry of multiple zeta values", Proceedings of the International Congress of Mathematicians, 2, (2006), 627-636.
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