**Master programme on**
**"Mathematics and Applications"**
Department of Mathematics (UAM)
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**Advanced Course in Statistics**
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## Scope and objectives

The course is devoted to some topics of "infinite-dimensional statistics". This expression refers to those subjects of mathematical statistics in which the sample data and/or the target parameter are elements of an infinite-dimensional space. The book by Grenander (1981) is a classical pioneering reference where, under the title *Abstract Inference*, the problems of infinite dimensional statistics are presented in a unified framework. The oldest and best known precedent of this type of infinite-dimensional problems is maybe the theory of **nonparametric functional estimation** whose development started at the early 1950's and reached its height in the late 1990's. This theory is concerned with the estimation of functions (typically the density function and the general regression curve) and, more recently, the estimation of sets (distribution supports or density level sets). In all cases the available sample information consists of random observations in the real line or the Euclidean space. Therefore, in this case the sample space is finite-dimensional but the target parameter is a member of an infinite-dimensional space.

On the other hand, in the early 1990's started a quick development of the **statistical methodology for functional data**, often called "functional data analysis" (FDA). This theory deals with those statistical problems in which the sample data are functions and, therefore, "live" usually in an infinite-dimensional space. The progress of this theory has been boosted by strictly practical motivations related with the increasing availability of functional (continuous-time) data in different fields of science and technology.

The purpose of this course is to offer an up-to-date overview of both, the theory of nonparametric functional estimation (density, regression and set estimation) and the statistical theory for functional data. A particular attention will be paid to those aspects which currently present a more complete mathematical elaboration. The computational aspects will be tackled via the popular software **R**.

Contents

**1. Nonparametric functional estimation and its applications**

1.1 Nonparametric estimation of the density and the regression function. Some important classes of estimators: basic concepts, examples, asymptotic properties.

1.2 The problem of choice of the smoothing parameters.

1.3 Set estimation: support and level set estimation. Some basic results. Applications.

1.4 Computational issues: **R** software.

**2. An introduction to the statistical methodology with functional data**

2.1 General setup. Examples. Longitudinal data and functional data.

2.2 Exploratory analysis of functional data. Data depth measures.

2.3 Regression and analysis of variance with functional data. Functional principal components methodology.

2.4 The problem of supervised classification (discrimination): statement and basic results. Main methods for the functional case. Main differences with respect to the analogous problem with finite-dimensional data.

2.5 The problem of non-supervised classification (clustering) with functional data.

2.6 Computational issues: functional data analysis in **R**.

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