Master programme on
"Mathematics and Applications"
Department of Mathematics (UAM)
Academic Year 2010-2011

Course on Numerical Methods
Tutor: Julia Novo

Scope of the course

The course is devoted to the study of different numerical methods to approximate the solution of partial differential equations. We will first concentrate on finite difference methods and their application to standard model problems. This will allow us to present the methods using simple terms but at the same time treating important and general concepts as stability and convergence with a reasonable degree of mathematical rigor. Then, we will study finite element methods. These methods represent a powerful and general class of techniques for the approximate solution of partial differential equations. We will provide an introduction to their mathematical theory, with special emphasis on theoretical questions such as accuracy, reliability and adaptivity; practical issues concerning the development of efficient finite element algorithms will also be discussed. Finally, we will concentrate on spectral methods giving an introduction to this kind of methods that were originally proposed in the 1944 as a tool for large-scale computations in fluid dynamics. Along the course we will show different examples using MATLAB and the students will program some simple codes to better understand the theoretical concepts.

Contents

2.6 Introduction to numerical fluids dynamics: Stokes and Navier-Stokes equations.

2.7 A posteriori error estimation techniques.

2.8 Stabilized methods for advection-diffusion problems.

3. Introduction to spectral methods.

3.1 General questions.

3.2 Periodic problems. Fourier methods.

3.3 Non-periodic problems. Legendre and Chebyshev methods.

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