Master programme on
"Mathematics and Applications"
Department of Mathematics (UAM)
Academic Year 2010-2011

Advanced Course in Analysis: Calderón-Zygmund Theory, Weights and Elliptic Operators
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Scope of the Course

In this course we will start with the classical Calderón-Zygmund theory. We will further study operators associated to the Kato problem which may not be representable with "usable" kernels. The lack of smoothness on the kernels makes the classical theory unapplicable and we will need to develop a generalized Calderón-Zygmund theory in absence of kernels. One of the main tools for those results is the representation of the operators in question in terms of a semigroup which satisfies "off-diagonal" estimates. We will pay special attention to the estimates with Muckenhoupt weights both from the classical point of view and also with the recent extrapolation results that enrich this theory.

Contents

1.Introduction:

1.1 Lebesgue spaces. Weak type inequalities.

1.2 Interpolation.

1.3 Decreasing rearrangements.

1.4 Function spaces.

1.5 BMO.

2.Calderón-Zygmund Theory:

2.1 Hardy-Littlewood maximal function.

2.2 Calderón-Zygmund decomposition.

2.3 Hilbert transform. Calderón-Zygmund operators.

2.4 Sharp maximal function.

3.Weighted norm inequalities:

3.1 A_p condition. Reverse Hölder inequality.

3.2 The class A_∞. Factorization.

3.3 Rubio de Francia extrapolation.

3.4 Weighted strong and weak type inequalities for Calderón-Zygmund operators.

3.5 Good-λ inequalities. Applications.

4.Generalized Calderón-Zygmund theory and elliptic operators:

4.1 Kato problem.

4.2 Semigroup, Riesz transforms, functional calculi and square functions associated to elliptic operators. "Off-diagonal" estimates.

4.3 Calderón-Zygmund theory for singular "non-integral" operators.

Bibliography

1. Auscher, P.On necessary and sufficient conditions for L^p-estimates of Riesz transforms associated to elliptic operators on R^n and related estimates. *Mem. Amer. Math. Soc.*, 186(871), 2007.

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4. Grafakos, L. *Classical Fourier Analysis*, volume 249 of *Graduate Texts in Mathematics*. Springer, New York, 2nd edition, 2008.

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