Master programme on
"Mathematics and Applications"
Department of Mathematics (UAM)
Academic Year 2010-2011

Advanced Course in Geometry
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Scope of the course

The course is an introduction to the part of Compact Riemann Surfaces Theory needed to understand Belyi's Theorem and the Grothendieck Theory of Dessins d'enfants, which is the final goal. It includes in particular:

- Equivalence between compact Riemann surfaces, Fuchsian groups and complex algebraic curves.
- Exposition Of Belyi's Theorem, which characterizes those compact Riemann surfaces that can be defined over a number field as the covers of the sphere ramified above three values (*Belyi functions*)
- Correspondence between Belyi functions and certian knd of graphs embedded in compact topological surfaces (*dessins d'enfants*).
- Action of the absolute Galois group on dessins.
- Explicit examples.

Prerequisites

Knowledge at the level of a Degree in Mathematics about Galois Theory, complex variables, group theory and the fundamental group of a surface.

Contents

1. Riemann surfaces and algebraic curves

1.1 Basic definitions. Morphisms. Differentials.

1.2 Topology of Riemann surfaces.

1.3 Algebraic curves, function fields and Riemann surfaces.

2. Riemann surfaces and Fuchsian groups

2.1 Uniformization.

2.2 Existence of meromorphic functions.

2.3 Fuchsian groups. Triangle groups.

2.4 Automorphisms of Riemann surfaces.

2.5 Monodromy.

3. Belyi's Theorem

3.1 Belyi's contribution.

3.2 Algebraic characterization of morphisms. Galois action.

3.3 Riemann surfaces and valuations. Galois action on points.

3.4 The criterion of definability over $\overline{\mathbf{Q}}$.

4. Dessins d'enfants

4.1 First definitions. Permutation representation of a dessin.

4.2 Dessins d'enfants and Belyi pairs.

4.3 Fuchsian group description of dessins. Regular and uniform dessins.

4.4 The action of $\mathrm{Gal}(\overline{\mathbf{Q}})$ on dessins. Faithfulness of the action.

4.5 Examples.

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