Master programme on
"Mathematics and Applications"
Department of Mathematics (UAM)
Academic Year 2010-2011

Advanced Course in Partial Differential Equations
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Scope and objectives

The primary aim of the course is to introduce the students to different techniques to solve nonlinear elliptic and parabolic equations arising is physical models, classical and new. The models considered are elementary but paradigmatic in the theory, giving behaviors that could be find in more involved and general problems. Mainly problems of reaction-diffusion, of growth and related with optimal transport will be considered. As a second goal of the course we will try to show to the students basics methods in the mathematical research activity, for instance to find and handle bibliographical information necessary to get knowledge, preparation of technical reports etc.

Contents

1. An elliptic problem in divergence form

1.1 Existence of weak solutions.

1.2 Summability of the weak solution with respect to the summability of the data. Stampacchia's results.

1.3 The De Giorgi result

1.4 Leray-Lions operators

2. Variational problems

2.1 Eigenvalues of linear elliptic equations.

2.2 Minimization of functional.

2.3 Ambrosetti-Prodi problems.

2.4 Ekeland variational principle and critical point theory: mountain pass theorem.

2.5 Critical problems: Pohozaev identity.

2.6 The concentration-compactness methods. Theorem of Brezis-Nirenberg.

3. Solutions of infinite energy

3.1 Existence of solution for linear equations with data $f \in L^m(\Omega)$, $1 \leq m < \frac{2N}{N+2}$. Entropy solution, solution obtained as a limit of approximation.

3.2 The equation $-\Delta u = |\nabla u|^2 + f$, stationary model of Kardar-Parisi-Zhang. Related equations.

3.3 Necessary and sufficient conditions for existence.

3.4 Non-uniqueness: the Laplace equation with measure data.

4. Some parabolic problems

4.1 Semi-linear Cauchy problems for the heat equation.

4.2 Comparison principles.

4.3 Local existence in time.

4.4 The Fujita phenomenon: blow-up in finite time.

4.5 Global existence for small data.

4.6 The Kardar-Parisi-Zhang growth model.

5. Some problems limit of PDE respect to parameters

5.1 Preliminaries and models.

5.2 Basic estimates and passage to the limit.

5.3 The Δ_∞ operator: Viscosity solutions.

5.4 Some results and the relation with the Monge-Kantorovich framework of optimal transport.

Bibliography

Books

1. Ambrosetti, A.; Malchiodi, A. ; *Nonlinear analysis and semilinear elliptic problems.* Cambridge Studies in Advanced Mathematics, 104. Cambridge University Press, Cambridge, 2007.

2. Ambrosetti, A.; Malchiodi, A. ; *Perturbation methods and semilinear elliptic problems on* R^n. Progress in Mathematics, 240. BirkhÃuser Verlag, Basel, 2006.

3. Ambrosetti, A. ; Prodi, G. ; *A primer of nonlinear analysis.* Cambridge Studies in Advanced Mathematics, 34. Cambridge University Press, Cambridge, 1995.

4. Aubin, T. ; *A course in differential geometry.* Graduate Studies in Mathematics, 27. American Mathematical Society, Providence, RI, 2001.

5. Brezis, H. ; *Analyse fonctionelle. Théorie et Applications*, Masson, 1983.

6. Caffarelli, L. A.;Cabré, X. ; *Fully Nonlinear Elliptic Equations* American Mathematical Society, Colloquium Publications Vol 43, 1995.

7. Dacorogna, B.; *Direct Methods in the Calculus of Variations*, Springer-Verlag, 1989.

8. Evans, L. C ;*Partial differential equations.* Graduate Studies in Mathematics, 19. American Mathematical Society, Providence, RI, 1998.

9. Evans, L. C.; Gariepy, R. F.; *Measure theory and fine properties of functions.* Studies in Advanced Mathematics. CRC Press, Boca Raton, FL, 1992.

10. Evans, L. C.; *Weak convergence methods for nonlinear partial differential equations.* CBMS Regional Conference Series in Mathematics, 74. Published for the Conference Board of the Mathematical Sciences, Washington, DC; by the American Mathematical Society, Providence, RI, 1990.

11. Giusti, E. ; *Metodi Diretti nel Calcolo delle Variazioni*, U.M.I. 1994.

12. Quittner, P.; Souplet, P. ; *Superlinear parabolic problems. Blow-up, global existence and steady states.* BirkhÃuser Advanced Texts, BirkhÃuser Verlag, Basel, 2007.

13. Vazquez, J. L.; *The Porous Medium Equation. Mathematical Theory* Clarendon Press, Oxford, 2007.

14. Villani, CÃ©dric; *Optimal transport. Old and new.* Grundlehren der Mathematischen Wissenschaften , 338. Springer-Verlag, Berlin, 2009.

15. Villani, CÃ©dric; *Topics in optimal transportation.* Graduate Studies in Mathematics, 58. American Mathematical Society, Providence, RI, 2003.

Articles

Some articles will be proposed to the students. The study and exposition of these papers will be used to evaluate to the students.