

**Master programme on
“Mathematics and Applications”**
Department of Mathematics (UAM)
Academic Year 2010-2011

Algebraic curves

Tutor: Ana Bravo

SCOPE OF THE COURSE

The course is an introduction to the basics of Algebraic Geometry. Special attention will be paid to the theory of algebraic curves. In particular elliptic curves will be studied as a prerequisite to the course on Criptography to be taught on the second semester.

CONTENTS

1. Introduction: Algebraic varieties and function fields

- 1.1 Affine algebraic varieties. Coordinate rings.
- 1.2 Noetherian rings. Hilbert's basis Theorem.
- 1.3 Hilbert's Nullstellenzat.
- 1.4 Morphisms and rational functions.

2. Algebraic projective curves

- 2.1 Algebraic functions on a projective curve.
- 2.2 Valuations.
- 2.3 Singularities.
- 2.4 Blow-up.

3. Riemann-Roch Theorem

- 3.1 Divisors.
- 3.2 Riemann's Theorem.
- 3.3 The Canonical Divisor. The genus of a curve.
- 3.4 Riemann-Roch's Theorem.

4. Some results about curves

- 4.1 Hurwitz's Theorem.
- 4.2 Embeddings in the projective space. The canonical embedding.
- 4.3 Hyperelliptic curves.
- 4.4 The degree of a curve. Classification of curves.

5. Elliptic curves

- 5.1 The group law.
- 5.2 Classification of elliptic curves. The j -invariant.

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5. Liu, Q. *Algebraic Geometry and Arithmetic Curves* (Oxford Graduate Texts in Mathematics) Oxford University Press, New edition (2006).
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