PARTIAL DIFFERENTIAL EQUATIONS

Master Course   First Quarter, 2010-11

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Program

Stationary Block: Second-order elliptic equations
1.- Laplace's equation. Classical boundary-value problems. Elementary analysis. The Maximum Principle. 2.- Perron's method. 3.- Problem of the minimum of a functional: Variational calculus. 4. The Sobolev spaces. 5. Weak solutions. The Lax-Milgram Theorem. 6.- Eigenvalue problems.
Complements: 7.- The Obstacle Problem in Elasticity. Free Boundary Problems in Mechanics. 8.- A priori estimates, existence and regularity. 9. Nonlinear elliptic equations.

Evolution block. Parabolic equations
1.- The heat equations: elementary properties. 2.- The heat equation as a continuous limit of a random walk. Finite differences. 3.- Generation of semigroups. 4.- Asymptotic bahaviour. 5.- Nonlinear diffusion equations.

Complements: 6.- Reaction-diffusion equations. 7.- Schrödinger's equation. 8.- Stokes's equation. 9.- Scalar conservation laws.

Bibliography. Reference text :

[Ev] Evans, L. C. (1998). *Partial Differential Equations*, Graduate Studies in Mathematics, Vol.19, AMS.

Other references:
[GT] Gilbarg, D., Trudinger, N. S. *Elliptic partial differential equations of second order.* Reprint of the 1998 edition. Classics in Mathematics. Springer-Verlag, Berlin, 2001.
[J] John, Fritz, *Partial differential equations.* Reprint of the fourth edition. Applied Mathematical Sciences, 1. Springer-Verlag, New York, 1991.
[S] Strauss, Walter, *Partial differential equations. An Introduction,* Wiley, 1992.

Objetives. The course is aimed at graduate students with an understanding of the basic theory of differential equations. It focusses on a medium-to-advanced level of the theory of elliptic and parabolic equations, with extension to more advanced material or to other equations in the complements.

Expected performance of the students: they should become familiar with a wide class of techniques and also a wide number of results, both classical and more recent, and be able to perform by themselves proofs of variations of the main results. The listed complements are possible avenues for further study or for final course works.

Methodology. The theory and problems of the course will be evaluated through partial exams and a main final exam. The practical weekly work will be taken into account, in particular oral expositions of selected topics or problems. A final course work in the form of a personal memoir will be important in assessing the grades of the more advanced students.

Practical implementation of the classes: two sessions per week, 90 minutes each. Personal tutor sessions as required.

Class material (texts, notes, problem sheets, lectures or events) will be announced via the web page of the course and distributed as convenient.