Master programme on
"Mathematics and Applications"
Department of Mathematics (UAM)
Academic Year 2010-2011

Differential Geometry
Professor: Luis Guijarro

Course description:

Differential geometry deals with smooth manifolds, high dimensional locally euclidean objects with an structure that allows the appearance some of the main objects of differential calculus.

This is an introductory class in some of the main topics of differential geometry. The student should have had some previous exposition to the elementary concepts of manifolds, vector fields, smooth functions and smooth applications at an undergraduate level. Our aim is to develop some of the main ideas required to help the student reach a point where he can start an independent study of differential geometry oriented towards a master's thesis.

Needless to say, some of the topics mentioned in the program should be covered briefly due to time requirements.

Contents

3. Lie groups and Lie algebras

3.1 Definitions. Invariant vector fields.

3.2 Subgroups and subalgebras.

3.3 The adjoint representation.

4. Riemannian Geometry

4.1 Definitions and examples of Riemannian metrics.

4.2 Affine connections and covariant derivatives. The Levi-Civita connection.

4.3 Geodesics.

4.4 The curvature operator. Curvature identities. Sectional, Ricci and scalar curvatures.

4.5 Jacobi fields and the exponential map.

4.6 Variation formulas. Some global theorems: Hopf-Rinow, Cartan-Hadamard, and Bonnet-Myers.

Bibliography

1. Boothby, William M., *An introduction to differentiable manifolds and Riemannian geometry.* Academic Press, (2003).
2. Do Carmo, Manfredo., *Riemmanian Geometry*, Birkhauser (2002)
3. Kobayashi, S. y Nomizu, K., *Foundations of differential geometry*, John Wiley and Sons, (1996).
4. Poor, W., *Differential geometric structures.* McGraw-Hill, 1981, or Dover (2007).
5. Sakai, T., *Riemmanian Geometry*, AMS (2005).
6. Spivak, M., *A comprehensive introduction to differential geometry.* Publish or Perish, (1999).
7. Walschap, G., *Metric structures in differential geometry.* Springer Verlag, (2004).