

DEPARTMENT OF MATHEMATICS
UNIVERSIDAD AUTÓNOMA DE MADRID

Graduate course:

FOUNDATIONS OF MATHEMATICAL ANALYSIS

Objectives: The course offers a panoramic view of the different areas of Mathematical Analysis (Real and Complex Function Theory, Operator Theory, Functional Analysis, etc) illustrating its basic character and its interaction with other areas of Mathematics.

Prerequisites: The very basics of Measure Theory, Real, Complex and Functional Analysis.

PROGRAM

1. Measures and Lebesgue spaces.
2. Real variable techniques for convergence problems (approximation, interpolation, maximal functions, etc.)
3. Complex variable techniques (factorization, univalent functions, normal families, etc)
4. Functional Analysis techniques (Hilbert and Banach spaces, duality, etc.)
5. The Dirichlet problem (harmonic and subharmonic functions, Perron's method)
6. Integral equations and elements of the spectral theory of operators.
7. Distribution Theory and Sobolev spaces.

Bibliography:

- L.V. Ahlfors: *Complex Analysis*, McGraw - Hill, New York 1985.
J.B. Conway: *A Course in Functional Analysis*, Springer 1985.
R.G. Douglas: *Banach Algebra Techniques in Operator Theory*, Springer 1998.
R. E. Edwards: *Functional Analysis: Theory and applications*. Dover, 1995.
G. Folland: *Real Analysis. Modern Techniques and Their Applications*, John Wiley & Sons, Inc., 1999.
J. García-Cuerva and J.L. Rubio de Francia: *Weighted Norm inequalities and related topics*. North Holland, 1985.
P. D. Lax: *Functional Analysis*. John Wiley & sons, 2002.
E.H. Lieb y M. Loss: *Analysis*, GSM 14, AMS, Providence, R.I., 2001.
W. Rudin: *Real and Complex Analysis*, McGraw-Hill, New York, 1987.
W. Rudin: *Functional Analysis*, McGraw-Hill, New York, 1973.
V.A. Sadvnichii: *Theory of Operators*, Kluwer Academic Publishers, Nueva York y Londres, 1991.
E.M. Stein & R. Shakarchi: *Real Analysis: Measure Theory, Integration, and Hilbert Spaces*, Princeton University Press, 2005.
R.J. Zimmer: *Essential Results of Functional Analysis*, The Univesity of Chicago Press, Chicago y Londres, 1990.

Comments: 1.-In the program above, the content of each chapter is not fully developed. The concrete development will ultimately depend on the previous background of the students. In an ideal situation, the students should be very familiar with the basic notions of Real, Complex and Functional Analysis and that would allow to cover the material with great depth and amplitude. In less than ideal situations, some time must be devoted to review the basics and some students could be even advised to take some additional course at the undergraduate level to improve their background.

2.- In any case, not all the subjects should be necessarily fully covered in the lectures. An important goal of the course is to train the students to write and present mathematics in a clear and professional way. This can be achieved by leaving some subjects to be worked by the students as individual research projects at the end of the course. These projects, handed as written papers and presented in front of the class in sessions that could include questions and discussions by the instructor and the fellow students, are an important part of the course and contribute, together with the homework assignments of exercises, to be handed in writing and also presented on the blackboard, to the final grade of the course.