**Master programme on
"Mathematics and Applications"**
Department of Mathematics (UAM)
Academic Year 2010-2011

Fluid Mechanics
Tutor: Diego Córdoba Gazolaz

SCOPE AND OBJECTIVES

The course is devoted to the Euler and Navier-Stokes equations. After a physical introduction of this equations, we will study the local existence of solutions, some criteria which provide global existence and the existence of weak solutions.
The purpose is to give an introduction to fluid mechanics in order to understand some open problems which are currently under research.

CONTENTS

1. The Euler and Navier-Stokes equations

1.1 Physical introduction.

1.2 Conservation Law and general characteristics.

1.3 A brief introduction to singular integral operators.

1.4 The pressure and the vorticity.

2. The Quasi-Geostrophic equation

2.1 General characteristics.

2.2 QG and 3D Euler.

3. Well-posedness

3.1 A brief introduction to the Sobolev spaces

3.2 Energy estimates.

3.3 The particle-trajectory method.

4. Global existence

4.1 Global existence in two dimensions.

4.2 Beal-Kato-Majda theorem.

4.3 Singularities in 1D models.

5. Weak solutions

5.1 Introduction to weak solutions.

5.2 Vortex Patch and Vortex Sheet.

5.3 The α-patchs, the Muskat problem and Water Waves.

5.4 Global existence of weak solutions in two dimensions.

5.5 Global existence of weak solutions for QG.

5.6 Non-uniqueness for the Euler equation.

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