Master programme on
"Mathematics and Applications"
Department of Mathematics (UAM)
Academic Year 2010-2011

Stochastic processes
Tutor: Javier Cárcamo

Scope of the course

The aim of the course is to provide a general perspective of the most important stochastic processes and their applications. Due to the largeness of the subject, we do not focus on details and we avoid highly technical proofs. We concentrate on the motivation of the results and the ideas behind the main concepts.
We start with some introductory lectures to explain the basic concepts, the differences between previous courses in probability and the new subjects and some examples of important processes. Afterwards, we analyze Markov chains (in discrete and continuous time), martingales and their applications to ruin problems, the Brownian motion and, finally, stochastic integration and applications to financial models.

Contents

4. Conditional expectation and martingales in discrete time

4.1 Conditional expectation.

4.2 Definition of a martingale. Basic properties.

4.3 Optional stopping theorem.

4.4 Some results about convergence of martingales.

4.5 Comparison methods for stochastic models.

4.6 Strassen's theorem.

5. Brownian motion. Applications

5.1 Definition and motivation.

5.2 Basic properties.

5.3 Martingales in continuous time.

5.4 Martingales associated to the Brownian motion.

5.5 Applications.

6. Stochastic integration and Itô's formula

6.1 Definition.

6.2 Basic properties.

6.3 Itô's formula and applications.

6.4 Black-Scholes's models.

Bibliography

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