The Fuglede conjecture and Riesz bases of exponentials.

8 HOURS COURSE

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Tentative schedule: Tuesday, June 23, 10:00 - 12:00 (Seminario 320); Thursday June 25, 10:00 - 12:00 (Seminario 420); Tuesday, June 30, 11:00 - 13:00 (Seminario 320); Thursday, July 2, 11:00 - 13:00 (Seminario 320).

Course outline:

1. A simple proof of Fuglede Conjecture for lattices ([1]):

1. An introduction to the Fuglede Conjecture
2. Lattices and their dual lattices
3. A simple proof of equivalence of tiling and exponential orthonormal basis

2. Exponential Riesz bases, Paley-Wiener spaces, and the correlation ([3])

1. Exponential Riesz bases
2. Paley-Wiener spaces
3. Necessary and sufficient condition for exponential Riesz bases
4. Existence of Riesz basis of vector valued functions

In 1974, Bent Fuglede conjecture that a domain Ω in $\mathbb{R}^d$ admits an orthogonal exponential bases $\{e^{2\pi ib\cdot}\}_{b\in B}$ if and only if $\mathbb{R}^d$ can be tiled by a family of translates of Ω. Flugede ([2]) proved this result when B is a lattice in $\mathbb{R}^d$. A proof of this result due to A. Iosevich ([1]) will be given in the first part of the course.

The second part of the seminar will be dedicated to relate Riesz bases of exponentials for $L^2(\Omega)$ to similar properties of the translation systems in the Paley-Wiener space $PW_\Omega = \{f \in L^2(\mathbb{R}^d) : \widehat{f}(\xi) = 0 \text{ a.e. } \xi \in \Omega^c\}$.

References

[1] Alex Iosevich, *Fuglede conjecture for lattices*, preprint.

[2] Bent Fuglede, *Commuting self-adjoint partial differential operators and a group theoretic problem*, J. Funct. Anal. 16 (1974), 101-121.

[3] Alex Iosevich, Azita Mayeli, *Exponential bases, Paley-Wiener spaces and applications*, Journal of Functional Analysis, Volume 268, Issue 2, 2015, Pages 363-375.