ON MEAN LIPSCHITZ SPACES

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Abstract. The analytic mean Lipschitz space $\Lambda(p,\alpha)$ is usually defined as the collection of functions f in the Hardy space H^p such that

$$\|f^*(\theta - t) - f^*(\theta)\|_p \leq C|t|^\alpha,$$

for all small t, where f^* is the boundary function. A theorem of Hardy and Littlewood says that $f \in \Lambda(p,\alpha)$ if and only if the integral means of the derivative satisfy

$$M_p(r, f') = \mathrm{O}\left((1-r)^{\alpha-1}\right), \quad r \to 1^-.$$

We show that the condition

$$\|f_r - f\|_p = \mathrm{O}((1-r)^\alpha), \quad r \to 1^-,$$

where $f_r(z) = f(rz)$ are the dilations of f, is equivalent to the Hardy-Littlewood condition, and discuss some analogues for Bergman and other spaces.